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Research article

The modified extended tanh-function method and its applications to the generalized KdV-mKdV equation with any-order nonlinear terms

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Abstract

In this article we apply the modified extended tanh-function method to find the exact traveling wave solutions of the generalized KdV-mKdV equation with any order nonlinear terms. This method presents a wider applicability for handling many other nonlinear evolution equations in mathematical physics.

Keywords: The generalized KdV-mKdV equation with any order nonlinear terms; the modified extended tanh–function method; Riccati equation; Traveling wave solutions.

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INTRODUCTION

The investigation of the traveling wave solutions of nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In recent years, new exact solutions may help us to find new phenomena. A variety of powerful methods, such as the inverse scattering method [1, 13], the bilinear transformation [7], the tanh-sech method [10, 11], the extended tanh method [10], the homogeneous balance method [5] and the Jacobi elliptic function method [15] were used to develop nonlinear dispersive and dissipative problems. The pioneer work of Malfliet in [10, 11] introduced the powerful tanh method for reliable treatment of the nonlinear wave equations. The useful tanh method is widely used by many authors such as [17–20] and the references therein. Later, the extended tanh method, developed by Wazwaz [21, 22], is a direct and effective algebraic method for handling nonlinear equations. Zayed et al [23] have used this method to find the exact solutions of the (2+1)-dimensional Nizhnik-Novikov-Veselov equations and the (1+1)-dimensional Jaulent-Miodek (JM) equations.

The objective of this paper is to apply the modified extended tanh-function method to find the exact traveling wave solutions of the generalized KdV-mKdV equation with higher-order nonlinear terms [24] in the form

$$u_t + (\alpha + \beta u^p + \gamma u^{2p})u_x + u_{xxx} = 0, \qquad (1)$$

where α, β, γ are constants and $p \neq 0$. Li [25] has discussed Eq. (1) using the (G'/G)-expansion method and found its exact solutions, while Zayed et al [26] have applied the two variable $(G'/G, 1/G)$-expansion method and determined the exact solutions of the combined KdV-mKdV equation. The paper is organized as follows: In Sec. 2, the modified extended tanh-function method is presented. In Sec. 3, we use the modified extended tanh-function method to obtain exact solutions of the generalized KdV-mKdV equation with any-order nonlinear terms. Sec. 4, some conclusions are given.

DESCRIPTION OF THE MODIFIED EXTENDED TANH-FUNCTION METHOD

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0, \qquad (2)$$

Where F is a polynomial in $u(x,t)$ and its partial derivatives. In the following we give the main steps of this method.

Step 1. We use the wave transformation

$$u(x,t) = u(\xi), \quad \xi = x - ct, \qquad (3)$$

where c is a constant, to reduce Eq. (2) to the ODE:

$$P(u, -cu', u', c^2 u'', ...) = 0, \qquad (4)$$

where P is a polynomial in $u(\xi)$ and its total derivatives, such that $' = d/d\xi$
.

Step 2. We suppose that the solution of Eq. (4) has the form

$$u(\xi) = a_0 + \sum_{i=1}^{m} (a_i \phi^i + b_i \phi^{-i}), \qquad (5)$$

where a_i, b_i are constants to be determined, such that $a_m \neq 0$ or $b_m \neq 0$ and ϕ satisfies the Riccati equation

$$\phi' = b + \phi^2, \qquad (6)$$

where b is a constant. Eq. (6) admits several types of solutions:

(i) If $b < 0$, then

$\phi = -\sqrt{-b}\tanh(\sqrt{-b}\,\xi)$, or $\phi = -\sqrt{-b}\coth(\sqrt{-b}\,\xi)$,

(ii) If $b > 0$, then

$\phi = \sqrt{b}\tan(\sqrt{b}\,\xi)$, or $\phi = -\sqrt{b}\cot(\sqrt{b}\,\xi)$,

(iii) If $b = 0$, then

$\phi = -\frac{1}{\xi}.$

Step 3. We determine the positive integer m in (5) by balancing the highest order derivatives and the nonlinear terms in Eq. (4). In some nonlinear equations the balance number m is not a positive integer. In this case, we make the following transformations:

$$u(\xi) = v^{\frac{q}{p}}(\xi), \tag{7}$$

where $m = \frac{q}{p}$ is a fraction in the lowest terms. Substituting (7) into (4) to get a new equation in the new function $v(\xi)$ with a positive integer balance number.

Step 4. We substitute (5) along with Eq. (6) into Eq. (4) and collecting all the terms of the same power $\phi^i, i = 0, \pm 1, \pm 2, \ldots$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple to get the values of a_i, b_i, and c.

Step 5. Substituting these values and the solutions of Eq. (6) into (5) we have the exact solutions of Eq. (2).

APPLICATIONS

In this section, we will apply the method described in Sec.2 to find the exact traveling wave solutions of the generalized KdV-mKdV equation with any- order nonlinear terms (1). To this end, we use the wave transformation (3) to reduce Eq. (1) to the following ODE:

$$u''' + (\alpha + \beta u^p + \gamma u^{2p})u' - cu' = 0. \tag{8}$$

By balancing u''' with $u^{2p}u'$ in Eq. (8), we get $m = \frac{1}{p}$. According to step. 3, we make the transformation

$$u(\xi) = v^{\frac{1}{p}}(\xi), \tag{9}$$

where $v(\xi)$ is a new function of ξ. Substituting (9) into Eq. (8), we get the new ODE

$$p^2 v^2 v''' + (\alpha + \beta v + \gamma v^2 - c) p^2 v^2 v' + 3p(1-p) v v' v'' + (2p^2 - 3p + 1) v'^3 = 0. \tag{10}$$

Balancing $v^2 v'''$ with $v^4 v'$ in Eq.(10) gives $m = 1$. Consequently, we get the solution

$$v(\xi) = a_0 + a_1\phi + b_1\phi^{-1}, \tag{11}$$

where a_0, a_1, b_1 are constants to be determined, such that $a_1 \neq 0$ or $b_1 \neq 0$. Now, substituting (11) along with equation (6) into (10) , collecting the coefficients of ϕ^i and setting them to zero, we get a system of algebraic equations for a_0, a_1, b_1 and c . Using the Maple, we get the following results.

Case 1.

$$a_0=\frac{-\beta(2p+1)}{2\gamma(p+2)},\quad a_1=0,\quad b_1=\pm\frac{\beta^2p(2p+1)}{4\gamma(p+2)^2}\sqrt{\frac{-(2p+1)}{\gamma(p+1)}},\ b=\frac{\beta^2p^2(2p+1)}{4\gamma(p+1)(p+2)^2},$$
$$c=\frac{p^3\alpha\gamma+5p^2\alpha\gamma+8p\alpha\gamma+4\alpha\gamma-2p\beta^2-\beta^2}{\gamma(p+1)(p+2)^2}. \tag{12}$$

Form (11), and (12), we deduce the traveling wave solutions of Eq.(1) as follows:
For $b<0$, we obtain the solution

$$u(\xi)=\left[\frac{-\beta(2p+1)}{2\gamma(p+2)}\left(1\pm\coth\left(\sqrt{-b}\,\xi\right)\right)\right]^{\frac{1}{p}}, \tag{13}$$

For $b>0$, we obtain the solution

$$u(\xi)=\left[\frac{-\beta(2p+1)}{2\gamma(p+2)}\left(1\pm i\,\cot\left(\sqrt{b}\,\xi\right)\right)\right]^{\frac{1}{p}}, \tag{14}$$

where $\xi=x-\frac{p^3\alpha\gamma+5p^2\alpha\gamma+8p\alpha\gamma+4\alpha\gamma-2p\beta^2-\beta^2}{\gamma(p+1)(p+2)^2}t$.

Case 2.

$$a_0=\frac{-\beta(2p+1)}{2\gamma(p+2)},\quad a_1=\pm\frac{1}{p}\sqrt{\frac{-(2p^2+3p+1)}{\gamma}},\quad b_1=0,\ b=\frac{\beta^2p^2(2p+1)}{4\gamma(p+1)(p+2)^2},$$
$$c=\frac{p^3\alpha\gamma+5p^2\alpha\gamma+8p\alpha\gamma+4\alpha\gamma-2p\beta^2-\beta^2}{\gamma(p+1)(p+2)^2}. \tag{15}$$

In this case, we deduce the traveling wave solutions of Eq.(1) as follows:
For $b<0$, we obtain the solution

$$u(\xi)=\left[\frac{-\beta(2p+1)}{2\gamma(p+2)}\left(1\pm\tanh\left(\sqrt{-b}\,\xi\right)\right)\right]^{\frac{1}{p}}, \tag{16}$$

For $b>0$, we obtain the solution

$$u(\xi)=\left[\frac{-\beta(2p+1)}{2\gamma(p+2)}\left(1\pm i\,\tan\left(\sqrt{b}\,\xi\right)\right)\right]^{\frac{1}{p}}, \tag{17}$$

where $\xi=x-\frac{p^3\alpha\gamma+5p^2\alpha\gamma+8p\alpha\gamma+4\alpha\gamma-2p\beta^2-\beta^2}{\gamma(p+1)(p+2)^2}t$.

Case 3.

$$a_0=\frac{-\beta(2p+1)}{2\gamma(p+2)},\quad a_1=\pm\frac{1}{p}\sqrt{\frac{-(2p^2+3p+1)}{\gamma}},\quad b_1=\pm\frac{\beta^2 p(2p+1)}{16\gamma(p+1)(p+2)^2}\sqrt{\frac{-(2p^2+3p+1)}{\gamma}},$$
$$b=\frac{\beta^2p^2(2p+1)}{16\gamma(p+1)(p+2)^2},\quad c=\frac{p^3\alpha\gamma+5p^2\alpha\gamma+8p\alpha\gamma+4\alpha\gamma-2p\beta^2-\beta^2}{\gamma(p+1)(p+2)^2}. \tag{18}$$

In this case, we deduce the traveling wave solutions of Eq.(1) as follows:
For $b<0$, we obtain the solution

$$u(\xi)=\left[\frac{-\beta(2p+1)}{2\gamma(p+2)}\pm\frac{\beta(2p+1)}{4\gamma(p+2)}\left(\tanh\left(\sqrt{-b}\,\xi\right)+\coth\left(\sqrt{-b}\,\xi\right)\right)\right]^{\frac{1}{p}}, \tag{19}$$

For $b>0$, we obtain the solution

$$u(\xi)=\left[\frac{-\beta(2p+1)}{2\gamma(p+2)}\pm\frac{\beta(2p+1)i}{4\gamma(p+2)}\left(\tan\left(\sqrt{b}\,\xi\right)+\cot\left(\sqrt{b}\,\xi\right)\right)\right]^{\frac{1}{p}}, \tag{20}$$

where $\xi=x-\dfrac{p^3\alpha\gamma+5p^2\alpha\gamma+8p\alpha\gamma+4\alpha\gamma-2p\beta^2-\beta^2}{\gamma(p+1)(p+2)^2}t.$

CONCLUSIONS

In this article, the modified extended tanh-function method was applied to give the traveling wave solutions of the generalized KdV-mKdV equation with any order nonlinear terms. On comparing our solutions (13)- (20) with that obtained in [25], we have many new solutions using the proposed method of this paper which are equivalent in some cases and not in other cases. The performance of this method is reliable and effective and can be applied to many other nonlinear evolution equations. Finally, we note that the method obtained in [21,22] is called the extended tanh-function method which is absolutely different from the proposed method used in the present article.

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