

**Research article**

# **MATHEMATICAL MODEL OF HYDROCARBON TRANSPORT INFLUENCED BY HYDRAULIC CONDUCTIVITY AND POROSITY OF SOIL IN PORT HARCOURT, NIGER DELTA OF NIGERIA**

**Eluozo. S. N**

Subaka Nigeria Limited Port Harcourt; Civil and Environmental Engineering Consultant Department of Research and Development, Rivers State of Nigeria.  
E-mail: [solomoneluozo2000@yahoo.com](mailto:solomoneluozo2000@yahoo.com)

---

## **Abstract**

Hydrocarbon transport are influenced by some characteristics in soil and water environment, this is one of the major causes of soil degradation in deltaic environment, these type of pollution are caused by so many ways, the activities of man has develop hydrocarbon pollution in deltaic environment. The soils become infertile and developed low nutrient and bear capacity stability to withstand an imposed load in structures, including the pollution of aquiferous zone. Rapid transports of these contaminants are caused by different factors. To monitor the rate of hydrocarbon influenced by porosity and hydraulic conductivity, mathematical model were developed to monitor the behaviour of hydrocarbon at different formation to ground water aquifers the system developed considered the dependent and independent variables, expressing the variables through mathematical tools, an equation were developed and derived considering every condition that will definitely influence the system, the system considered different phase of the soil deposition, including the contaminants behaviour mixing up with water on the soil, the developed model also considered the geological deposition that influence the stratification of the soil. The rate of porosity and hydraulic conductivity were the independent variables considered, because it plays major roles in the transport of these contaminants. The model developed expressed the conditions that affect the rate of contaminants; the model will definitely monitor the rate of hydrocarbon under the pressure of hydraulic conductivity and degree of porosity of the soil at different formation to ground water aquifers. **Copyright © IJESTR, all rights reserved.**

**Keywords:** mathematical model, hydrocarbon influenced on porosity and hydraulic conductivity

---

## **1. Introduction**

The studies of hydraulic conductivity are determined by the soil structural deposition. This influence of stratification is where variable that influence the soil matrix is expressed. The study of hydrocarbon transport spread on the soil causing environmental pollution is the focus of the study. The pollution sources are caused by some influences, which could be manmade activities or natural origin, the rate of oil exploration, without environmental risk assessment, and environmental statement are the major risk hazard, the stipulated environmental laws should as a baseline in solving environmental related problems caused of manmade activities [Eluozo 2012].

The transport and fate of groundwater contaminants is controlled by advective transport, dispersion, sorption to aquifer sediments, biological degradation, and other processes. Spatial variations in groundwater velocity are recognized to provide the dominant control on contaminant transport in most settings (Brusseau, 1994). Groundwater velocity is proportional to hydraulic conductivity, which may vary by several orders of magnitude across small spatial ranges as a result of the geological processes that deposited and altered the sediments. Solutes follow the path of highest hydraulic conductivity, thus it is critical to identify these regions, as well as the low conductivity barriers that impede flow. Heterogeneity in hydraulic conductivity can be evaluated using a variety of methods including tracer tests, slug and pump tests, permeameter measurements on soil core samples, and borehole flow meter tests. Rehfeldt et al. (Rehfeldt et al., 1992) reported that hydraulic conductivity varied by 4 orders of magnitude over a 240 m long and 10 m deep region at Columbus Air Force Base in northeastern Mississippi using a borehole flow meter. Even in a localized area, significant variability has been observed, such as the order of magnitude variations observed across a 19 m long and 2 m deep portion of the Borden site (Canadian Forces Base, Ontario) (Sudicky, 1986). Sorption processes also exert significant control over transport, but spatial variations in sorption properties are often neglected because they are cumbersome to measure. Most groundwater modeling activities use average values for sorption parameters, such as the retardation coefficient, and these are normally based on either a limited number of measurements or estimates from bulk soil samples. Variations in sorption capacity can be especially important in the design of in situ remediation systems such as the biocurtain that was recently installed at the Schoolcraft site in Michigan (Dybas et al., 2002; Hyndman et al., 2000).

While the spatial variability of sorption parameters (Pickens et al., 1981; Elabd et al., 1986; Mackay et al., 1986; Rao et al., 1986; Wood et al., 1987) and hydraulic conductivities (Sudicky, 1986; Rehfeldt et al., 1992) have been documented separately, very few studies have investigated whether they are correlated. Robin et al. (Robin et al., 1991) determined the distribution coefficients for strontium using aquifer material from the Borden site. Vertical and horizontal variability of distribution coefficients and hydraulic conductivity were studied along two orthogonal horizontal transects. They found very weak negative correlation between the two parameters. Because organic and inorganic chemicals are sorbed by different mechanisms, we have no theoretical basis to extrapolate these findings to organic contaminants (Xianda, et al 2005). Contaminated soil and groundwater poses a serious problem with respect to soil / ground water quality and the risk of spreading pollutants into other compartments of the environment. The major concern at most contaminated sites is the risk of groundwater pollution by organic and inorganic compounds (Grathwohl and Klenk, 2000). Since the remediation of all contaminated sites is economically not feasible, ground water risk assessment procedures are needed for the ranking of sites, decision making on future use and remedial actions. At sites where petroleum products are handled or stored, contamination of the unsaturated

soil zone is frequently found. Hydrocarbons can reach the ground water by transport with percolating water and by spreading in the soil gas. Degradation process can limit the spreading of the contaminants.

The vadose zone usually consists of a heterogeneous geologic medium that provides crucial pollution protection to the groundwater through various physical, chemical, and biological processes. Vadose zone investigation provides valuable information regarding the source, extent, and strength of subsurface contamination, its (potential) impact on groundwater, and implications for remediation, such as evaluating the need and adequacy of certain remedial actions. Findings from vadose zone investigations also have important regulatory ramifications for identifying sources of groundwater contamination. Due to the importance of groundwater as a natural resource, a large number of studies on subsurface contamination were performed in the last decade. It became clear that there are significant uncertainties in the study of subsurface contaminations, especially those by toxic organic chemicals such as chlorinated solvents. These studies also highlighted the multidisciplinary nature of this particular environmental problem. There are a number of recent reviews on this topic, each with different emphases (e.g., Jury and Flühler, 1992; Nielsen *et al.*, 1986; Schwille, 1984; USEPA, 1989,1990, 1991, Samuel and Yu 1994).

## 2. Theoretical background

Hydraulic conductivity is the rate at which soil can hold on fluid, when the degree of saturation is very high, the rate of fluid increases to the extent that these saturation rate is high. Such soil hold water at high volume, the rate of hydraulic conductivity of the soil is determined based on the geological formation of the study area. As a deltaic environment, the rate of hydraulic conductivity and porosity of the stratum are very high, this condition allow for fast migration of contaminant from every source. The study focused on hydrocarbon, the contaminant expedited and distributed for refunding through distribution network of the pipe; if the pipe develops crack and spill form those leakages from the pipe. Hydrocarbon will continue to transport and pollute the soil, the study area is found to have a lot of hazards from the hydrocarbon spillage, and the concepts were developed by [20], the concept were modified to suite the condition of the deltaic environment in the study location. The influences of the fast transport of the hydrocarbon are considered as a variable, the variables are denoted through the application of mathematical tools that form the system (Eluozo, 2012) . The variables are as follows

## 3. Governing equation

### Nomenclature

$\phi$	=	Soil porosity [-]
$S_o$	=	Saturation of oil [-]
$h_o$ [ L ]	=	Water height equivalent pressure for phase p oil given by $h_p = \rho_p / \rho_w$
$R_o$	=	Net mass transfer per unit porous media volume for oil [ M/L <sup>3</sup> ]
$U_z$	=	Unit gravitational vector, measured positive upward [ - ]

- $Z$  = Elevation [ L ]  
 $T$  = Time [ T ]  
 $Ko$  = Phase conductivity tensor for oil [L/T].  
 $V$  = Velocity of transport  
 $P$  = Density of water [MLT<sup>-2</sup>]

$$\phi ho \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial Z} \left[ Koho \left[ \frac{\partial S_o}{\partial Z} + PROU_z \right] \right] + \frac{\partial}{\partial Z} \left[ Koho \left[ \frac{\partial S_o}{\partial Z} + PROU_z \right] \right] + \frac{Ro}{Po} \dots (1)$$

Substituting  $S_o = ZT$  into equation (1)

This equation was linearized in other to simplify the solution considering all the variables in the system, denoted using mathematical tools. In other to express there relationship that influence hydrocarbon transport and pollute the soil. The following expressions are applied: Substituting  $h_w = ZT$  into equation (1)

$$\phi ho Z^1 T = Z \left[ Koho \left[ Z^1 T + PROU_z \right] \right] + Z \left[ Koho \left[ Z^1 T + PROU_z \right] \right] + \frac{Ro}{Po} \dots (2)$$

$$\phi ho \frac{T^1}{T} = \left[ Koho \left[ \frac{Z^1}{Z} + PROU_z \right] \right] + \left[ Koho \left[ \frac{Z^1}{Z} + PROU_z \right] \right] + \frac{Ro}{Po} \dots (3)$$

$$\phi ho \frac{T^1}{T} = \left[ Koho + PROU_z \right] \frac{Z^1}{Z} + Koho \frac{Z^1}{Z} + \left[ \frac{Koho + PROU_z}{ZT} \right] \dots (4)$$

$$\phi ho \frac{T^1}{T} = PROU_z \frac{Z^1}{Z} + Koho + Koho \frac{Z^1}{Z} + Koho PROU_z + \frac{Ro}{Po} \dots (5)$$

$$\phi ho \frac{T^1}{T} = Koho + Koho PROU_z + \frac{Ro}{Po} = \lambda^2 \dots (6)$$

$$\phi ho \frac{T^1}{T} = \lambda^2 \dots (7)$$

$$Koho = \lambda^2 \dots (8)$$

$$\phi ho = \lambda^2 \dots (9)$$

This implies that equation can be expressed as:

$$Koho PROU_z + \frac{Ro}{Po} = \lambda^2 \dots (10)$$

The equation denote other parameters that express the rate of conductivity tensor for oil weight equivalent pressure head for oil, including gravitational acceleration, net mass transfer per unit porous medium for oil are express in the system to determine there relation, including there reaction on the transport process under the influence of hydraulic conductivity. The rates of porosity in the stratification of the formation determine the behaviour of the hydrocarbon transport in soil and water environment.

$$K_{oho} \frac{dy}{dZ} = \lambda^2 \dots\dots\dots (11)$$

$$\frac{dy}{dZ} = \frac{\lambda^2}{\phi_{ho}} \dots\dots\dots (12)$$

$$dy = \left[ \frac{\lambda^2}{\phi_{ho}} \right] dZ \dots\dots\dots (13)$$

$$\int dy = \int \frac{\lambda^2}{\phi_{ho}} = dZ \dots\dots\dots (14)$$

$$dy = \frac{\lambda^2}{\phi_{ho}} dZ \dots\dots\dots (15)$$

$$\int dZ = \int \frac{\lambda^2}{\phi_{ho}} dZ \dots\dots\dots (16)$$

$$y = \frac{\lambda^2}{\phi_{ho}} Z + C_1 \dots\dots\dots (17)$$

$$\Rightarrow 0 = \frac{-\lambda^2}{\phi_{ho}} Z + C_1 \dots\dots\dots (18)$$

$$C_1 = \frac{-\lambda^2}{\phi_{ho}} Z \dots\dots\dots (19)$$

This expression is modernized, the variables in the structure that should have a connection with the independent variables. The major independent variables are the hydrocarbon contaminating the soil, and the porosity degree that pressure the rapid movement of hydrocarbon. Base on the soil, the derived model equation expresses at equation (19) is to express the influence of porosity on hydrocarbon with respect to distance, whereby the transport is obeying the law of plug flow, migrating at different formation under the pressure of porosity.

But for further expression of these parameters, there relationship in terms of variables functions at various condition are considered, quadratic equation were adopted to relate the other variables expression, and to fulfill these condition in the system, quadratic expression were mathematically articulated in this form.

Applying quadratic formulae, we have

$$X = \frac{-b \pm \sqrt{b^2 - 4aC}}{2a} \dots\dots\dots (20)$$

Where  $a = \frac{\lambda^2}{\phi h o}$ ,  $b = C_1$ .

$$X = \frac{-(C_1) \pm \sqrt{(C_1)^2 - 4 \left[ \frac{\lambda^2}{\phi h o} \right]}}{2C_1} \dots\dots\dots (21)$$

$$X = \frac{-C_1 + \sqrt{C_1^2 - 4C_1 \frac{\lambda^2}{\phi h o}}}{2C_1} \dots\dots\dots (22)$$

$$\frac{-C_1 + \sqrt{C_1^2 - \frac{4C_1 \lambda^2}{\phi h o}}}{2C_1} \dots\dots\dots (23)$$

$$X = \frac{-C_1 - \sqrt{C_1^2 - \frac{4C_1 \lambda^2}{\phi h o}}}{2C_1} \dots\dots\dots (24)$$

Subject equation (24) to the following boundary condition and initial values condition

$$t = 0, S_o = 0 \dots\dots\dots (25)$$

Therefore,

$$Z_{(z)} = C_1 e^{-M_1 z} + C_2 e^{-M_2 z} \dots\dots\dots (26)$$

$$= C_1 \cos M_1 z + C_2 \sin M_2 z \dots\dots\dots (27)$$

Solving equation (19) gives

$$C_1 = \frac{\lambda^2}{\phi h o} z$$

$$So(z,t) = \left( C_1 \cos M_1 \frac{\lambda^2}{\phi h o} z + C_2 \sin M_2 \frac{\lambda^2}{\phi h o} z \right) \dots\dots\dots (28)$$

But if  $z = \frac{v}{t}$

The model can be written as:

$$So = (z,t) = \left( C_1 \cos M_1 \frac{\lambda^2}{\phi h o} \frac{v}{t} + C_2 \sin M_2 \frac{\lambda^2}{\phi h o} \frac{v}{t} \right) \dots\dots\dots (29)$$

The model established is to monitor the rate of hydraulic conductivity influence the fast migration of hydrocarbon contaminant, the degree of hydraulic conductivity are one of the major determinant hydrocarbon transport in soil and water environment, more so, reciprocally carrying its natural structure deposition and decreasing it bearing capacity. But when hydrocarbon were introduced through pipe spillage from surface to organic soil, it migration base on the degree of micropoles between the soil, transportation of this pollutant begin to reduce the bearing capacity of the soil including its fertility and characteristics, the rate of contamination is determined through soil stratification, based on the soil matrix. It is known also to be influenced by other environmental factors e.g. climatic condition of the study area. The location under study is prone to high rain densities, the rate of hydraulic conductivity will rapidly increase due to high rain intensities. The conditions allow rapid migration of hydrocarbon deposition from the distribution network.

Hydrocarbon in this study is found to mix together with the fluid from the soil, and rapidly contaminated the entire soil in the study area. Other factors will definitely increase high rate of migration, the rate of diffusion and dispersion; these variables are some of the influences that cause increase in migration. The final model equation also considered these parameters that are insignificant influence on the system, but focus more on the variables that expresses the major problem in the system. Mathematical model expression considered the variables because the parameter incorporated in the system are parameters that influence the hydrocarbon, the hydraulic conductivity of the soil influence the pollutant to migrate from one region to another region under the law of plug flow system; this reduced the bearing capacity; fertility and minerals that will be good or productive to the country at large. Considering the required parameters that influence the system, various environmental factors established there

influence on hydrocarbon through the migration from one region to other regions in the soil formation. The expressed model developed will assess the hydrocarbon influence from hydraulic conductivity and degree of porosity soil in by multifold influenced of contaminated hydrocarbon; the model can be applicable for simulation and model validation.

#### **4. Conclusion**

porosity in soil and its rate of hydraulic conductivity soil assessment from hydrocarbon contaminant is a serious problem in the study; the deltaic environment is prone to spillage of hydrocarbon, through the pipeline Vedas, the high rate of sabotage in our economy development has caused a lot of hydrocarbon pollution in the deltaic environment, contaminant mixed with fluid that deposited on the soil will definitely generate high influence transport of the hydrocarbon by the soil structural deposition.

The denoted expression on the rate of conductivity tensor for oil weight equivalent, pressure head for oil, including gravitational acceleration, net mass transfer per unit porous medium for oil are expressed in the system to determine their relation, including their reaction on the transport process under the influence of hydraulic conductivity. These are expressed in the system to monitor the rate of hydrocarbon in soil and water environment.

The influence of the system considered these variables that generate fast diffusion, this includes that also increase high concentration of hydrocarbon on the soil formation, and such characteristics are the basic causes of contaminants at rapid rate. The developed model considered different conditions based on the geologic history of the study location, the study area has been suffering from infertility of the soil from pollution of hydrocarbon contaminating the groundwater aquifer, surface water like creeks are prone to serious environmental hazard emanating from hydrocarbon pollution. The developed models that considered the basic cause as independent variables and were thoroughly expressed in the system, deriving the mathematical equation, the model was developed. The model can be applied to monitor the rate of hydrocarbon contaminant in unsaturated soil, migrating through multifold in the soil and water environment.

#### **References**

- [1] Xianda Zhao Roger B. Wallace a, David W. Hyndman Michael J. Dybas Thomas C. Voice Heterogeneity of chlorinated hydrocarbon sorption properties in a sandy aquifer *Journal of Contaminant Hydrology* 78 (2005) 327–342.
- [2] Rao, P.S.C., et al., 1986. Spatial variability of pesticide sorption and degradation parameters. *ACS Symp. Series* 315, 100–115.
- [3] Rehfeldt, K.R., Boggs, J.M., Gelhar, L.W., 1992. Field study of dispersion in a heterogeneous aquifer 3. Geostatistical analysis of hydraulic conductivity. *Water Resour. Res.* 28 (12), 3309–3324.

[4] Robin, M.J.L., Sudicky, E.A., Gillham, R.W., Kachanoski, R.G., 1991. Spatial variability of strontium distribution coefficients and their correlation with hydraulic conductivity in the Canadian Forces Base Borden aquifer. *Water Resour. Res.* 27 (10), 2619–2632.

[5] Hyndman, D.W., Dybas, M.J., Forney, L., Heine, R., Mayotte, T., Phanikumar, M.S., Tataru, G., Tiedje, J., Voice, T., Wallace, R., Wiggert, D., Zhao, X., Criddle, C.S., 2000. Hydraulic characterization and design of a fullscale biocurtain. *Ground Water* 38 (3), 462– 474.

[6] Pickens, J.F., Jackson, R.E., Inch, K.J., Merritt, W.F., 1981. Measurement of distribution coefficients using a radial injection dual-tracer test. *Water Resour. Res.* 17 (3), 529– 544

[7] Dybas, M.J., et al., 2002. Development, operation and long-term performance of a full-scale biocurtain utilizing bioaugmentation. *Environ. Sci. Technol.* 36 (16), 3635– 3644.

[8] Elabd, H., Jury, W.A., Cliath, M.M., 1986. Spatial variability of pesticide adsorption parameters. *Environ. Sci. Technol.* 20 (3), 256– 260.

[9] Mackay, D.M., Ball, W.P., Durant, M.G., 1986. Variability of aquifer sorption properties in a field experiment on groundwater transport of organic solutes: methods and preliminary results. *J. Contam. Hydrol.* 1 (1–2), 119– 132.

[10] Wood, L.S., Scott, H.D., Marx, D.B., Lavy, T.L., 1987. Variability in sorption coefficients of metolachlor on a Captina silt loam. *J. Environ. Qual.* 16 (3), 251– 256.

[11] Robin, M.J.L., Sudicky, E.A., Gillham, R.W., Kachanoski, R.G., 1991. Spatial variability of strontium distribution coefficients and their correlation with hydraulic conductivity in the Canadian Forces Base Borden aquifer. *Water Resour. Res.* 27 (10), 2619–2632.

[12] Oluwapelumi.O.O Samuel.A.O (2010) Plume Behaviour for Petroleum Hydrocarbon in a Tropical Sand Tank: Laboratory Experiments and Scenario-Specific Modeling *European Journal of Scientific Research* Vol.39 No.4 pp.523-541

[13] Grathwohl, P. & I. Klenk, 2000. “Groundwater risk assessment at contaminated sites” (GRACOS). In: *Contaminated Soil 2000* vol. 2 (ed. By W. Harder et al.) pp831-834. Thomas Telford, Leipzig, Germany.

[14] Jury, W. A. and Flühler, H. 1992. Transport of chemicals through soil: mechanisms, models, and field applications. *Adv. Agron.* **47**:141–201.

[15], D. R., van Genuchten, M. Th., and Biggar, J. W. 1986. Water flow and solute transport processes in the unsaturated zone. *Water Resour. Res.* **22(9)**:89S–108S

[16] Schwille, F. 1984. Migration of organic fluids immiscible with water in the unsaturated zone. **In:**

[17] (Yaron, B. et al.,1994 *Pollutants in Porous Media, The Unsaturated Zone Between Soil Surface and Groundwater.* Berlin, Springer-Verlag

[18] USEPA. 1989. Seminar Publication, Transport and Fate of Contaminants in the Subsurface. EPA/ 625/4-89/019. September 1989

[19] USEPA. 1990. Handbook, Ground Water, Vol. 1, Ground Water and Contamination. EPA/625/6-90/ 016a. September 1990

USEPA. 1991. Ground Water Issue — Dense Nonaqueous Phase Liquids. EPA/540/4-91-002. March 1991

[20] Samuel C. T. Yu, D. Env. Transport and Fate of Chlorinated Hydrocarbons in the Vadose Zone — a Literature Review with Discussions on Regulatory Implications Journal of Soil Contamination, 3(4): (1994)

[21] Eluozo S.N (2012) Mathematical modeling of unsaturated soil multifold assessment on hydrocarbon contamination in coastal area of Okirika, Rivers State of Nigeria unpublished article in press